最適化を考慮した離散係数FIRフィルタの設計法

Design of FIR Filter with Discrete Coefficients considering Optimality

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【要　約】
情報化社会においては，大量の情報をコンピュータによってどう経営に生かすかということは，企業経営において重要な課題である．本研究では，情報技術の一つであるデジタルフィルタ設計問題を取り扱う．特に，符号付き2進数を係数に持つFIRフィルタ設計問題において，半無限計画法と分枝限定法を組み合わせたあらたな解法を提案する．これによって，従来は近似解しか得られなかったものが，厳密な意味での最適解を得ることが可能となった，また，計算機実験によって，本設計法の有効性についても検証したので，報告する．

キーワード： デジタルフィルタ，半無限計画法，線形計画法，離散計画法，分枝限定法，符号付き2進数

Abstract:
In this paper,we propose a new design method of FIR filters with Signed Power of Two (SP2) coefficients. In the method proposed here, the design problem of FIR filters is formulated as an discrete semi-infinite linear programming problem (DSILP), and the DSILP is solved using a branch and bound technique. We will guarantee the optimality of the solution obtained. Hence, it is possible to obtain the optimal discrete coefficients. It is confirmed that the optimal coefficients of linear phase FIR filter with the SP2 coefficients could be designed fast with enough precisions by the computational experiments.

Keyword:
digital filter, semi-infinite programming, linear programming, descrete programming,
branch and bound technique, signed power of two

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1 Introduction

In recent years tremendous advances have been achieved in computer hardware as well as in digital technology in general. Significant reductions in the cost, size, and power consumption of digital hardware have led to increasingly widespread application. Digital systems are finding their way into our lives in computers and communications. For many diverse applications, information is now most conveniently recorded, transmitted, and stored in digital form. As a result, digital signal processing (DSP) has become an exceptionally important modern tool.

Digital signal processing deals with the representation of signals as ordered sequencers of numbers and the processing of those sequences. Typical reasons for signal processing include: estimation of characteristic signal parameters, elimination or reduction of unwanted interference, and transformation of a signal into a form that is in some sense more informative.

For a signal to be completely representable and storable in a digital computer memory,it must be sampled in time and discretized in value.That is,it must be a practical digital signal with both finite duration and a finite number of discrete values. Very long sequences can be processed much at a time. To discretize the value, a rounding or quantization procedure must be used. However once sampled and converted to a fixed bit-length binary form, the signal data are extremely convenient. These data can be stored on hard disks or diskettes, on magnetic tape, or in semiconductor memory chips. All the advantages of digital processing are now available. Unfortunately, these signal data usually contain also noise data. To eliminate the noise data, we use the so called filter. There are at least two types of filters, that has finite impulse response (FIR filter) and infinite impulse response (IIR filter). Both filters are studied very deeply. In thesis paper,we deal with the FIR filter.

There are two methods for the realization of FIR filter, one is a software re-alization method and another is a hardware realization by using digital circuits.

In hardware implementation of FIR filters, the filter coefficients corresponding to multiplier coefficients are presented as the finite word length numbers. When the coefficients are simply rounded to the nearest discrete number, precision of filters are degraded from the one with the optimal real coefficients. There-fore, design methods of FIR filters with discrete coefficients have been widely researched [24], [33]. There are no design methods of designing filters that could be easily adapted to special design specifications. So each filter has to be de-signed, in principle, by a complete mathematical design procedure. It is the aim of all design methods to approximate a desired frequency response as close as possible by a finite number of FIR filter coefficients. The starting point of all these methods is the assumption of idealized frequency responses or tolerance specifications in the passband and stopband. Low variation of the magnitude (ripple) in the passband, high attenuation in the stopband and sharp cut-off are competing design parameters in this context. Some of error measures are generally used in FIR filter design. One is the average of the squared error in the frequency-response approximation. The second is the maximum of the error over specified regions of the frequency response and so on.The method based on the first error measure is called a least squared (LS) approximation, the second a Chebyshev approximation or equiripple approximation. And equi-ripple ap-proximation is much important since the characteristic of the response function is much better than the one obtained by the LS approximation.

Recently, many studies on a design method for linear phase FIR filters with discrete coefficients have been published [25], [29] in which,a numerical rep-resentation by a sum of signed power of two (SP2) has been used in several methods. [1],

[13], [28], [29]. It is a reason that a small number of non-zero digits is often required for a representation of the coefficients in a VLSI imple- mentation of the filters. There exist a lot of studies to obtain an approximated solution for this design problem. See, for example, Ito et. al [20], W. -S. Lu [25].

They proposed to use a semidefinite programming (SDP) relaxation method for the design problem. However, if we do not have the optimal solution for the de- sign problem, we cannot mention the performance of the approximation method precisely.

Since the design problem is formulated as a discrete semi-infinite linear pro-gramming problem, the most practical methods to solve the problem is to use the branch and bound (B & B) method.And,there are some methods using B & B method for the design problem, for example, based on LP, Remez algorithm, and so on. Cho et. al [15] proposed an B & B method based on LP focusing only on the active constraints to decrease the computational time. However, they did not assure the optimality of the solution obtained by the algorithm.

In this paper, we propose a new design method of linear phase FIR filters with SP2 coefficients which guarantees the optimality of the solution obtained. In the method proposed, the design problem is formulated as a discrete semi-infinite linear programming problem (DSILP) and solved by B & B method. In the B & B method,a branching tree is generated and, on each node, it is necessary to solve semi-infinite linear programming problem (SILP) [8].

It is shown by the results of some computational experiments for the filter designing problem, the developed algorithm is rather practical.

2 Problem Formulation

In this section, we introduce the design method of digital FIR filters with SP2 coe fficients.

2.1 Design problem of FIR digital filters with continuous coefficients

In this paper, we deal with a design problem of FIR digital filters with SP2 co-efficients that minimize the maximal error, i.e., minimize the following function:

$$e = \max_{\omega \in \Omega} |H(e^{j\omega}) - H_d(\omega)| \tag{1}$$

where $H_d(\omega)$ is the desired frequency response function and $\Omega = [0, \omega_p] \cup [\omega, \pi]$. Here, $[0, \omega_p]$ denotes a passband and $[\omega_s, \pi]$ denotes a stopband.

In the first, we consider the continuous coefficient case. Then the design function of the FIR filter is:

$$H(e^{j\omega}) = \sum_{k=0}^{N-1} h_k e^{-jk\omega}. \tag{2}$$

Now, we assume N is odd filter number. Given a budget of total number of power-of-two terms M, a certain number of SP2 terms, m_k , is allocated to the k-th target discrete-coefficient d_k. Then we denote the frequency response H $(e\,j^{\omega})$ as follows.

$$H(e^{j\omega}) = \sum_{k=0}^{N-1} d_k e^{-jk\omega} \tag{3}$$

The allocation of SP2 terms is determined, for example, by Lu [32], Ito et. al [19], [21].

We assume that the absolute value of each SP2 coefficient $\{d_k\}$ is in the in-terval $[2^0, 2^{-U}]$ where U is a natural number. Then, with a given term allocation m_k, the discrete coefficients d_k in the equation(6) can be expressed as,

$$d_k = \sum_{i=1}^{m_k} b_i^{(k)} 2^{-q_i^{(k)}}. \tag{4}$$

Since each SP2 coefficient d_k is consisted of m_k non-zero digits, the relation of m_0 , ... , m_k and M is represented as the following equation.

$$\sum_{k=0}^{N-1} m_k = M. \tag{5}$$

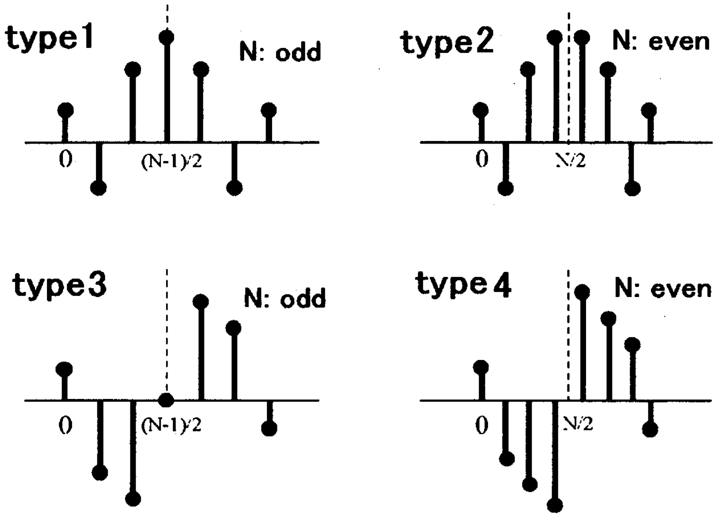


Figure 1: Four types of FIR filter

Here, we have $b_i^{(k)} \in \{-1, 1\}$ and $1 \leq q_i^{(k)} \leq U$, $(1 \leq i \leq m_k, 0 \leq k \leq N-1)$.

The coefficients of an FIR filter are easily constrained to produce a linear phase response. The corresponding constraint is simply that the finite-duration impulse response have even symmetry or odd symmetry about its midpoint. Linear phase FIR filter has an important property that the group delay is con-stant. The implication of constant group delay is that all frequency components of an input sequence are similarly delayed in the output sequence. The shapes of impulse response of FIR filter are classified into four types by filter length N and even or odd symmetry characteristic. These four cases are illustrated in Figure 1. In our proposed method, we consider FIR filter of type1 because it makes possible to design all types of filters (high-pass, low-pass and band-pass filters).

Since the impulse responses of all types are symmetry, the frequency responses are expressed as follows.

$$H(e^{j\omega}) = \sum_{k=0}^{N-1} d_k e^{-jk\omega} \tag{6}$$

$$= d_0 + d_1 e^{-j\omega} + d_2 e^{-2j\omega} \ldots d_{N-1} e^{-(N-1)j\omega} \tag{7}$$

$$= e^{-(N-1)/2j\omega} \sum_{k=0}^{(N-1)/2} d_k \cos k\omega \tag{8}$$

Omitting the linear phase factor $e^{-(N-1)/2j\omega}$, the frequency response of a sym-metrical impulse response filter with N odd is given by

$$H(\omega) = \sum_{k=0}^{K} d_k \cos k\omega. \tag{9}$$

Here $K = (N-1)/2$ and this equation is called a magnitude response. Then the number of filter coefficients we consider is $K+1$. Suppose a desired response $H_d(\omega)$ is given as follows

$$H_d(\omega) = \begin{cases} S, & \omega \in [0, \omega_p], \\ 0, & \omega \in [\omega_s, \pi]. \end{cases} \tag{10}$$

Where S is a scaling factor, ω_p is the passband cutoff frequency, and ω_s is the stopband cutoff frequency, respectively. Then, the optimal problem to approxi-mate $H(\omega)$ to $H_d(\omega)$ in a min-max sense can be written as

$$\min_{d_0,\ldots,d_K} \max_{\omega \in \Omega} |H(\omega) - H_d(\omega)|. \tag{11}$$

where $\Omega = [0, \omega_p] \cup [\omega_s, \pi]$. is the approximation band.

If we introduce a new variable δ that corresponds to the L_∞-approximation error, it is easy to convert the above min-max problem to the following mini-mization problem, that is a semi-infinite programming problem with SP2 coefficients (DSILP).

$$\begin{array}{lrcll} \min & \delta & & & \\ \text{sub.to} & H(\omega)+\delta & \geq & H_d(\omega), & \omega \in \Omega, \\ & -H(\omega)+\delta & \geq & -H_d(\omega), & \omega \in \Omega \end{array} \tag{12}$$

3 An algorithm for solving DSILP.

Our aim is to solve DSILP (12), but it is impossible to solve (11) directly. Hence, we solve SILP ignoring the constraints that each coefficient is an SP2. Then DSILP reduces to a standard SILP and we can use several standard methods to solve the SILP, see for example [8]. Since SILP is a continuous optimization problem, an obtained optimal solution does not always satisfy the condition that each coefficient is an SP2. Hence, we have to combine SILP and a B & B method.

If there are some $\bar{h}_i$'s that are not SP2, then select one $\bar{h}_j$ that is not a number with SP2 coefficients variable x_j and generate two subproblems, which one has an additional constraint $h_j \leq \lfloor h_j \rfloor$ and the other has an additional constraints $h_j \leq \lfloor \bar{h}_j \rfloor$. Here $\lfloor \bar{h}_j \rfloor$ is the maximum SP2 coefficients that is less than or equal to $\bar{h}_j$ and $\lfloor \bar{h}_j \rfloor$ is the minimum SP2 coefficients that is greater than or equal to $\bar{h}_j$.

To solve SILP problem,we exploited the 3-phase method, and we introduce the algorithm shortly in the following.

An algorithm for solving SILP

INPUT: $N, \omega_p, \omega_s, S, M, m_0, \ldots, m_K$
OUTPUT: $\bar{h}_0, \ldots, \bar{h}_K, \bar{\delta}$,
Phase 1:
Generate the discretized linear programming problem with discretizing parameter q.
Solve the discretized linear programming problem and obtain $\bar{h}\ \bar{\delta}\ \bar{y}, m_0, \ldots, m_K$
where $\bar{y}$ is an optimal dual variable vectors for the discretized linear programming problem and $m_0, \ldots, m_K$ is the frequencies that correspond to the active constraints in the descretized linear programming problem.

Phase 2:
Delete the variables $\bar{y}(\omega_i)$ that are zero and the corresponding ω_i from the solution for each pair $(\omega_i, \omega j)$ whose ω_i and ω_j are very close.
do
$y_\alpha(\omega_i) \leftarrow y_\alpha(\omega_i) + y_\alpha(\omega j)$, $\alpha = 1$ or 2,
$y_\alpha(\omega j) \leftarrow 0$,
$\omega_j \leftarrow (\omega_i + \omega j)/2$.
end of for
Phase 3:
Solve the LISP using Newton method or quasi Newton method with using
$(\bar{x}, \delta, \bar{y}, \omega_1, \ldots, \omega_i)$ as the initial solution.
Here, $\bar{y}$ and $\omega_1, \ldots, \omega_i$ are the variables left by the operation of phase 2.
Output the solution of the Newton/quasi Newton method.
Now, we describe the B & B method for solving DSILP in the following:

B & B procedure for DSILP:

INPUT: $N, \omega_p, \omega_s, S, M, m_0, \ldots, m_K$
OUTPUT: $h_0, \ldots, h_K, \delta$,
$k \leftarrow 0$,
$\bar{z} \leftarrow$ high value.
Generate DSILP (12), and set SILP $P(0)$ by relaxing the condition to be SP2 numbers.

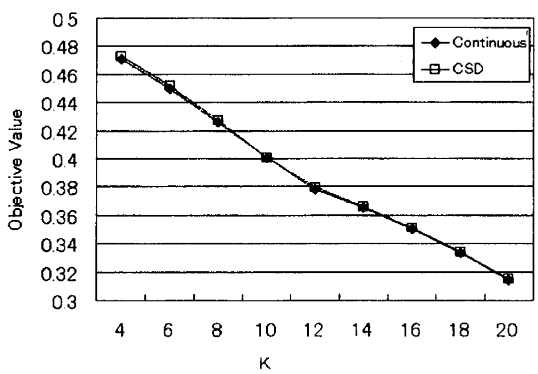
$\mathcal{P} \leftarrow \{P(0)\}$.
while $\mathcal{P} \neq \emptyset$ **do**
Select $P \in \mathcal{P}$.
$\mathcal{P} \leftarrow \mathcal{P} \setminus \{P\}$.
Solve SILP P by 3 Phase method.
if $\delta < \bar{z}$
then
if the optimal solution $(\bar{h}, \bar{\delta})$ of P is a solution with SP2 coefficients
then
$\bar{z} \leftarrow \bar{\delta}$,
$h^* \leftarrow \bar{h}$,
else
select j that $\bar{h}_j$ is not an SP2, and generate $P(k+1)$ by adding a constraint

$hj \geq \lceil \bar{h}_j \rceil$ to P,
generate $P(k+2)$ by adding a constraint
$hj \leq \lfloor \bar{h}_j \rfloor$ to P,
$\mathcal{P} \leftarrow \mathcal{P} \cup \{P(k+1), P(k+2)\}$,
$k \leftarrow k+2$.
end if
end if
end while
Output $h_0^*, \ldots, h_k^*, \bar{z}$.

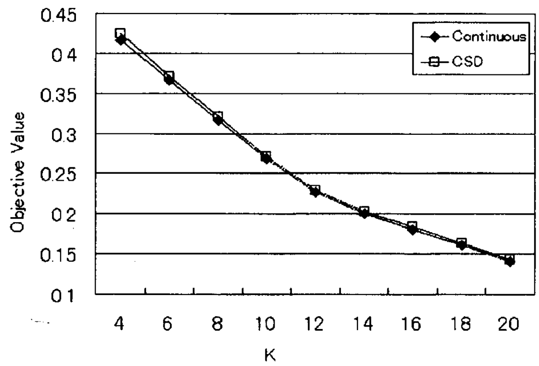
4 Numerical experiments

We executed some computational experiments to certify the performance of the proposed filter design method.

We consider a low pass filter with the odd length and the symmetric charac- teristic with $S = 1$.



(1) $\omega_p = 0.4\pi$, $\omega_s = 0.41\pi$



(2) $\omega_p = 0.4\pi$, $\omega_s = 0.43\pi$

Figure 2: Comparison of approximation errors

$$\Omega = [0, \omega_p] \cup [\omega_s, \pi],$$

$$H_d(\omega) = \begin{cases} 1, & 0 \leq \omega \leq \omega_p, \\ 0, & \omega_s \leq \omega \leq \pi, \end{cases} \quad (13)$$

where ω_p and ω_s are the pass and stop band cut-off frequencies, respectively.

The approximation errors from the proposed scheme are calculated for the following three sets of parameters, (A), (B), (C). $N = 9 \ldots 41$. Discretizing parameter q to generate the discretized linear programming problem is $4(K+1)$.

	M	ω_p	ω_s	U
(A)	$2(K+1)$	0.3π	0.35π	16,
(B)	$2(K+1)$	0.4π	0.41π	16,
(C)	$2(K+1)$	0.4π	0.43π	16,

We set each $m_k = 2$. The CPU is mobile Pentium III 650 MHz, and memory is 192 M bytes. We use glpk (Ver.4.4) [7] to obtain continuous solutions and to solve subproblems in Branch and Bound.The CPU time contains the execution time from the beginning to the end of obtaining the solution by our method.

In Figure 2 and Figure 3, we show the objective value of our method and of continuous solutions for $K = 4, 6, \ldots, 20$. The expression "Continuous" in Figure 3, Figure 2 means the optimal continuous solution and "CSD" means the CSD solution of our method.

In these figures, it was confirmed the objective

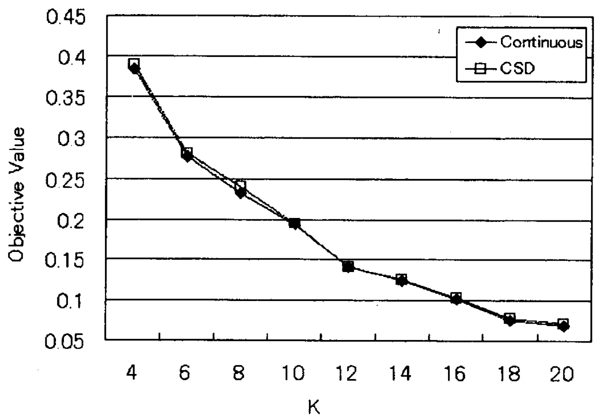
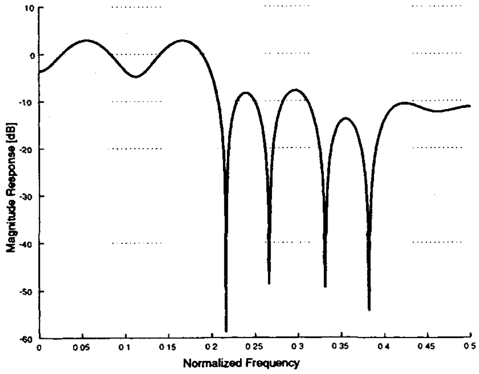
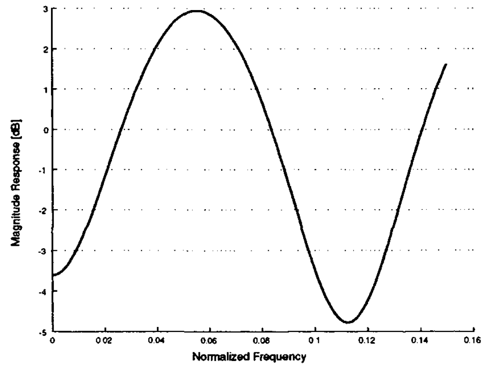
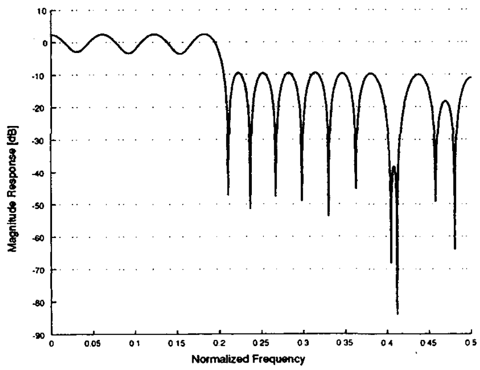


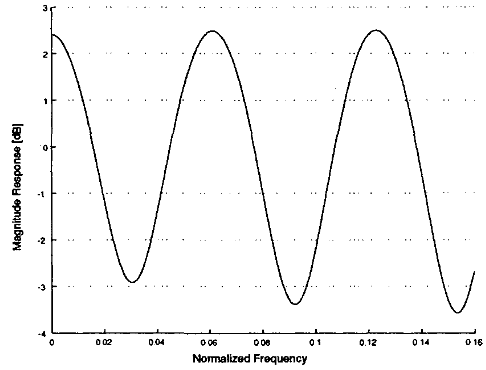
Figure 3: (3) $\omega_p = 0.3\pi$, $\omega_s = 0.35\pi$





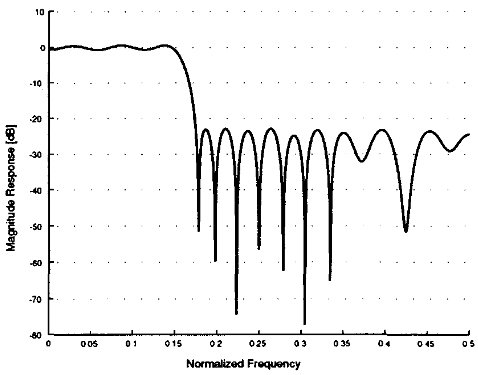
(1) $K = 14, \omega_p = 0.4\pi,\ \omega_s = 0.41\pi$ (2) $K = 14, \omega_p = 0.4\pi,\ \omega_s = 0.41\pi$

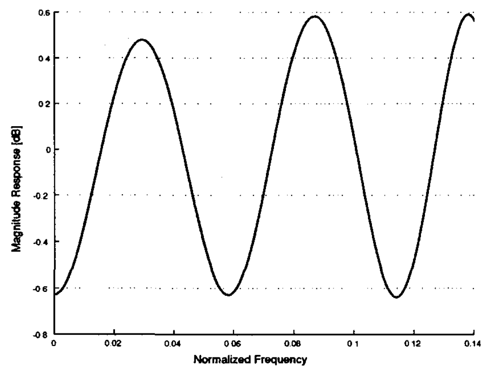




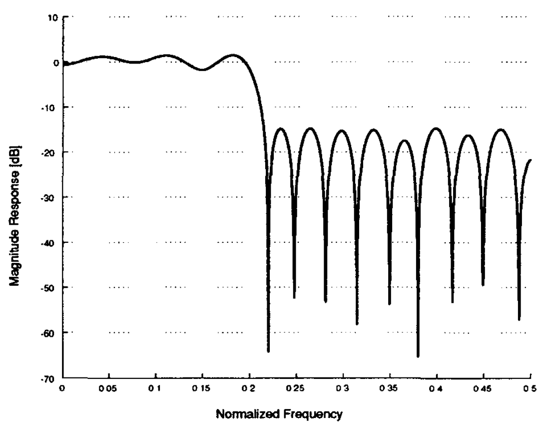
(3) $K = 18\omega_p = 0.4\pi,\ \omega_s = 0.41\pi$ (4) $K = 18\omega_p = 0.4\pi,\ \omega_s = 0.41\pi$

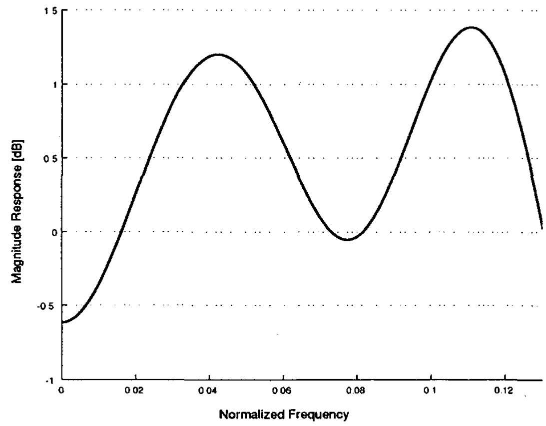
Figure 4: Magnitude response





(1) $K = 20, \omega_p = 0.3\pi,\ \omega_s = 0.35\pi$ (2) $K = 20, \omega_p = 0.3\pi,\ \omega_s = 0.35\pi$





(3) $K = 16, \omega_p = 0.4\pi,\ \omega_s = 0.43\pi$ (4) $K = 16, \omega_p = 0.4\pi,\ \omega_s = 0.43\pi$

Figure 5: Magnitude response

Table 1: Computational time (second)

K	A	B	C
4	9	5	1
6	23	7	2
8	44	13	27
10	219	122	82
12	309	155	290
14	495	442	656
16	2804	1924	2892
18	7799	4190	9340
20	25456	3654	14164

values of our method are close to that of optimal solutions. In general, it is known that the transferband gets narrow, it is difficult to design FIR filter, but in case of (1), the objective value by our method is almost optimal in spite that transferband is narrow. In Figure 4, the magnitude responses (9) are shown for $\omega_p = 0.4\pi$, $\omega_s = 0.41\pi$ and Figure 5 shows the magnitude responses for $\omega_p = 0.3\pi$, $\omega_s = 0.35\pi$ and $\omega_p = 0.4\pi$, $\omega_s = 0.43\pi$.

In Figure 4, it is observed that almost equi-ripple characteristic are obtained in both of two cases $K=14$, and $K=18$. Especially, in case of $K=18$, it is shown that the magnitude response is almost equi-ripple.

In Figure 5, these magnitude responses show that our method is efficient in not only stopband but also passband. In case of $K=14$, it is shown that the magnitude response in passband is small and in case of $K=16$, the magnitude response in stopband is almost qui-ripple.

In these results, it is shown that our method to design FIR filter is effect on obtaining of equi-ripple magnitude responses.

In the Table 1, the comparison of the computational time is shown.

As much as a K becomes big, calculation time grows large. However it is observed that the computation time of $K=20$ of (B) is about one hour,

5 Remarks

In this numerical experiments, we apply the discretizing parameter q is $4(K+1)$. According to this experiments, though we changed q from 4 ... 10 on conditions $K=4(A)$, $K=5(B)$, $K=6(C)$, the objective value did not change. It is possible to obtain better solution in discretizing much finer in other conditions. However it is confirmed that it is enough to discretize by $4(K+1)$ on the passband and stopband on the condition of numerical experiments to obtain optimal solutions.

6 Conclusion

In this paper, we propose a new design method of FIR filters with SP2 coeffi-cients. In this method, it is possible to obtain the optimal discrete coefficients.

It is confirmed that the optimal coefficients of linear phase FIR filter with the SP2 coefficients could be designed fast with enough precisions through the com-putational experiments.

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